Thursday 2-5

Determining the Angular Velocity of Winks using High Speed Video

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2.671 Measurement and Instrumentation
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Abstract

Tiddlywinks is a strategy game founded in 1955 in Cambridge University. Players, in singles or doubles, attempt to shoot plastic disks called "winks" into a plastic cup called the "pot". "Potting" winks can sometimes be difficult if the wink is "nurdled", which is when the wink is very close to the pot, typically beneath its upper rim. The lexicon of tiddlywinks terminology is exhaustingly extensive and can be found at www.tiddlywinks.org along with more game information.

In this experiment, we measured the angular velocity of a big wink being potted from a nurdled position and compared it to a far position approximately 4" away using high speed video. It was also determined if there was any deceleration during its flight. The results showed a statistically significant difference in angular velocity for the two cases. The angular velocity of the nurdled wink was 77.21 ± 2.56 rad/s and the angular velocity of the far wink was 83.69 ± 2.46 rad/s. Although both cases demonstrated a negative mean value for acceleration, they were not statistically significant from zero, indicating only minimal deceleration. These results are consistent with previous findings from experiments done with strobe photography.

1. Introduction

In this paper, we measure the angular velocity of a big wink when potted from a nurdled and far position. Potting is one of the most important aspects of tiddlywinks because winks in the pot are worth the most points at the end of the game. Thus, studying how winks behave when they are potted from different locations can provide insight into successfully performing these shots. The analysis of angular velocity is accomplished by analyzing high speed video of each shot using commercial software called LoggerPro. Furthermore, we determine the effect of frictions such as air currents on the wink by examining if there is a decrease in angular velocity during the course of the wink's flight.

In the nurdled situation, the wink is partially under the upper rim of the pot. Potting from this position typically requires a rapid wrist flick in order to spin the wink in such a way that it rotates over and above the rim of the pot. In the far situation, the wink is placed approximately 4" away from the pot, which is a common distance for potting. By realizing the actual behavior of the big wink in flight, tiddlywinks players can understand whether at a given distance they should focus on the rotational speed of the wink or some other factors such as the torque being applied and its angle.

This experiment extends the work done by Rick Tucker (MIT '80) in 1979 entitled "Tiddlywinks Photography"[1] in which he uses strobe photography to capture different shots at a maximum of 120fps. He provides qualitative assessments of the behavior of the shots and an approximation of the angular velocity of several potting shots. However, the only potting shot attempts are done with nurdled winks whereas this experiment also observes potting from afar. With advances in video technology, experiments can be performed using high speed video, which is significantly easier because it does not require the specific timing of photography to capture just the "right" moment.

2. Background

Tiddlywinks combines strategy elements of discrete games such as chess with manual skill that is required for making complex shots. There are four sets of colored winks and games can be played in pairs or singles, where opponents fight to pot their winks while covering their opponent's, which is called squopping. Potting winks scores points (but reduces the number of your winks in play) while squopping obstructs your opponent's ability to play his winks. When potting, a player attempts to flick a wink into the pot with a squidger, which is a larger plastic disk that strikes the wink. These elements of the game are shown in Figure 1 below.

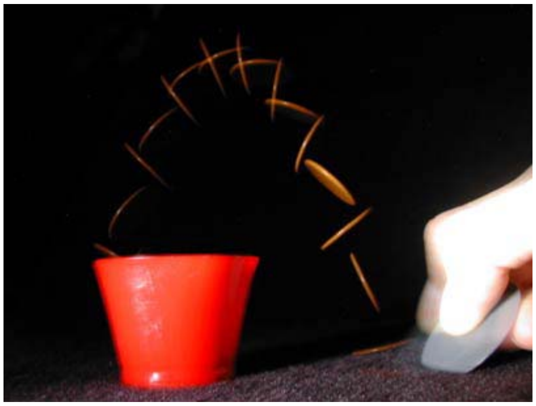

Figure 1: Strobe photograph of potting a big wink. The cup is called the pot and the large disk used to strike the wink is called a squidger (source: Robert Ochshorn).

Winks come in big and small varieties. They are 22mm and 16mm in diameter, respectively, and approximately 1.5mm thick according to "The Official Rules of Tiddlywinks"[2]. Each player starts with two big winks and four small winks. The pot is 38mm high with an external diameter of 48mm at the top and 38mm at the base. Squidgers are made out of plastic and must be between 25mm and 51mm in diameter and no thicker than 5mm at the edge. The game is played on a 3 feet by 6 feet mat.

2.1 Angular Velocity and Acceleration

To calculate the angular velocity of a wink about its center using vectors that describe the positions of its front (*f*) and back (*b*) lips, we define angles θ_0 and θ_1 as shown in Figure 2, which shows side views of a wink in two discrete time states. The position, (x, y), of the front and back lips are also shown. They are measured with respect to a common origin.

Figure 2: Angle and position of wink in two states with respect to the vertical.

Equation 1 defines θ_0 and θ_1 by the position of the front and back lips of the wink, where a positive angle is measured counter-clockwise from the vertical.

$$\theta_i = \tan^{-1}\frac{x_i^b - x_i^f}{y_i^f - y_i^b}, i = 0,1. \tag{1}$$

By definition, angular velocity is the change in angle per unit change in time. We report a clockwise change in angle as positive because the wink rotates in a clockwise direction in our videos. We can define ω, angular velocity, as the total change in angle divided by the change in time, which is shown in Equation 2:

$$\omega = \frac{\theta_0 - \theta_1}{t_1 - t_0}. \tag{2}$$

Angular acceleration, α, is the derivative of angular velocity. If we have angular velocities measured over time, the slope of the line relating angular velocity to time is the acceleration. Equation 3 defines acceleration as a function of angular velocity.

$$\alpha = \frac{\Delta\omega}{\Delta t}. \tag{3}$$

2.2 Verifying Camera Output with Gravity

The digital video file output of our Phantom 640 camcorder is 10 frames per second (fps) with an actual recording rate of 500fps. Thus, the file's timestamps must be multiplied by 1/50 to convert to the real time that is shown in each frame. This can be verified by performing a check on gravity using one of the videos.

Winks in flight can be described by a parabolic trajectory according to basic kinematics. Equation 4 defines the vertical position, y, of an object as a function of time:

$$y(t) = y_0 + v_0 t + \frac{1}{2} g t^2, \tag{4}$$

where y_0 is the initial height, v_0 is the initial velocity in the y direction, g is the acceleration due to gravity, and t is time.

If we fit a curve to vertical location of the wink in the digital video file, we should find that the constant in front of the t^2 term should be approximately $(1/2)(1/50)^2$ of the value of g (approximately 9.806 m/s^2).

3. Experimental Procedure

A standard big red wink was selected for the experiment and placed at a nurdled and far (approximately 4”) position for each video. The squidger used for potting the wink belonged to the author and is 44mm in diameter and of medium edge sharpness. A Phantom 640 digital camcorder was used to record the path of the wink into the pot at the following settings: 500fps, 200μs exposure time, 800x600 resolution, and non-interlaced mode. Two large halogen lamps were also used to provide extra lighting because the exposure time of each frame at high frame rates is very limited. The scale that we used for the analysis was the diameter of the pot, which is 51mm. Figure 3 shows a diagram and photo of our experimental setup.

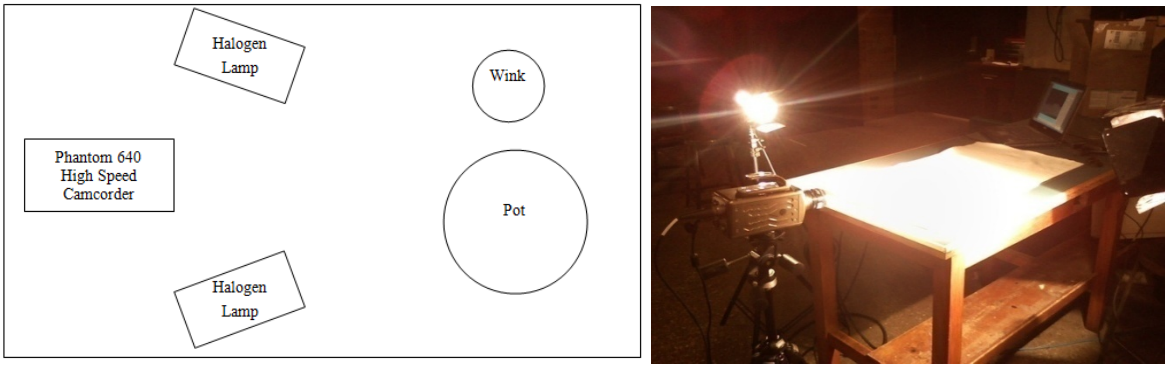

Figure 3: Schematic (not to scale) and photo of the experimental setup. The pot and wink cannot be seen in the photograph due to the overwhelming effect of the halogen lamps.

Samples from the videos are shown in Figure 4 for each of the positions that we successfully potted from. The far wink is on the left and the nurdled wink is on the right.

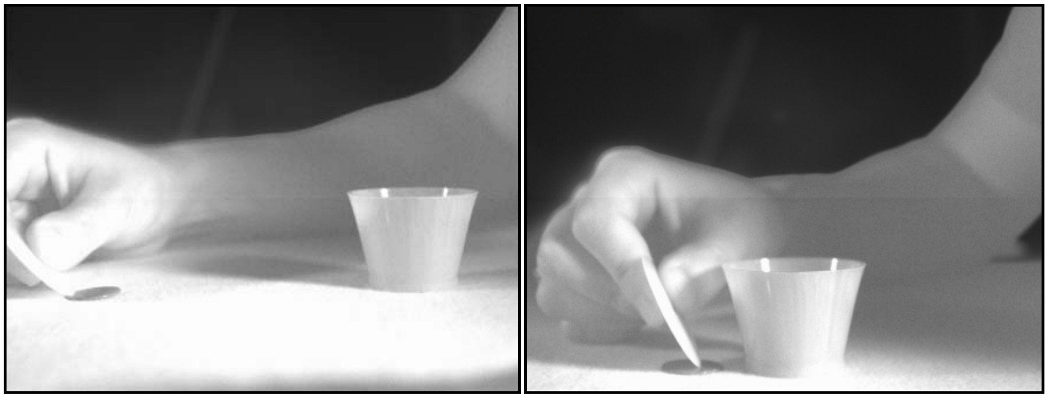

Figure 4: Screen capture from the Phantom 640’s video of the experimental setup immediately before potting in their original coloration.

A video clip was recorded of a successful pot from each position. This video was then imported into LoggerPro 3.5.0 for analysis using its movie feature. After defining an origin at the back lip of the wink, data points were selected for the positions of the back and front lips of the wink during its flight, which created the position vectors necessary for our measurements. Calculation of angular velocity and its error was performed in Microsoft Excel. MathCAD was used to determine the best fit line, its slope, and its error in order to determine if there were significant frictional forces acting on the wink.

Finally, a parabola was fit to the far wink's path with LoggerPro to check the value of g and verify that our time conversion was correct (see Section 2.2). This was done by taking the constant value in front of the t^2 term and multiplying by $2*50^2$ while accounting for the error. The value of gravity derived from this was $10.05 \pm 0.04\ m/s^2$, which is reasonably close to the accepted value of $9.806\ m/s^2$.

4. Results and Discussion

The angular velocity of the far wink was greater than that of the nurdled wink. Velocity was measured to be 83.69 ± 2.46 rad/s and 77.21 ± 2.56 rad/s, respectively. These values are the averages of the velocities over the course of the wink's flight. Angular acceleration, as determined by the slope of the best fit line, was -19.90 ± 52.34 rad/s^2 for the far wink and -36.53 ± 33.21 rad/s^2 for the nurdled wink. Although both values are negative and indicate deceleration, which is what we would expect, the confidence intervals are too large to make it statistically significant from zero or each other. These results are summarized in Table 1.

	Units	**Nurdled Wink**	**Far Wink**
ω	rad/s	77.21 ± 2.56	83.69 ± 2.46
α	rad/s^2	-36.53 ± 33.21	-19.90 ± 52.34

Table 1: Summary of the angular velocities and accelerations of the nurdled and far winks.

It is important to note that some data points were not used in the analysis. Any negative angular velocities were removed because it does not make sense for the rotation to instantly reverse direction. Extremely high and low angular velocities were also removed because it caused the data to be discontinuous. These factors resulted in the removal of nine points from the nurdled wink data set and six points from the far wink data set. Figures 5 and 6 show the resulting data and the best fit lines.

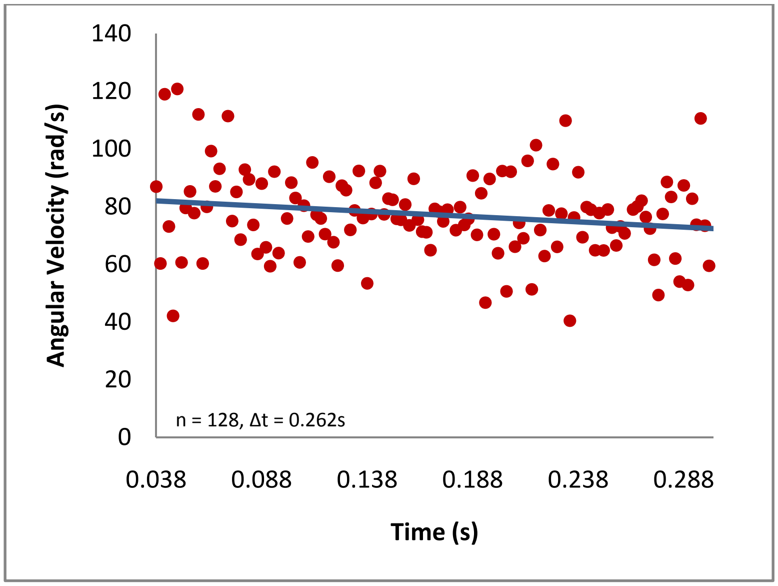

Figure 5: Angular velocity of nurdled big wink.

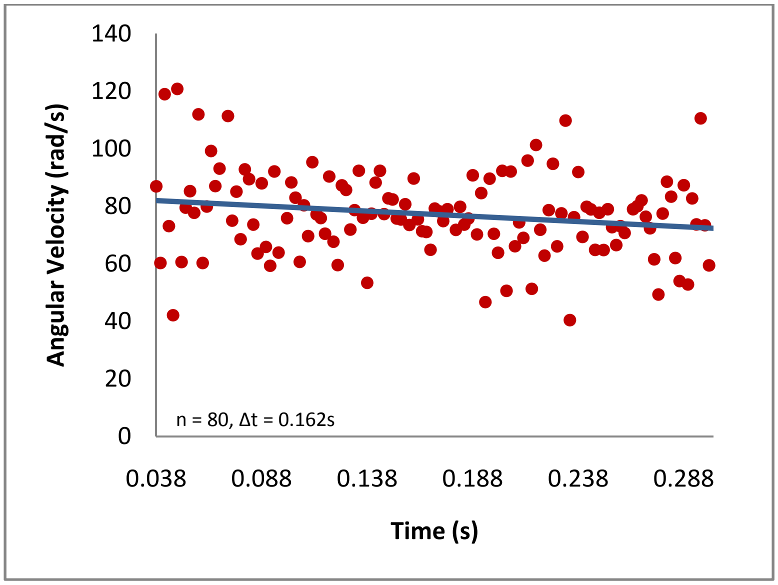

Figure 6: Angular velocity of far big wink.

Figures 5 and 6 show that the data had a great amount of variance in both cases, which results in very large error ranges for the slopes (acceleration). This difficulty arose from manually judging the position of the front and back lips in LoggerPro when constructing our data set. In the future, more lighting, a white background, and a white-sleeved shirt could improve the clarity of the wink in the video and yield better results for angular acceleration measurements. The error of the angular velocities was relatively low because of the law of large numbers. Because the sample size was large in both cases, the error decreased tremendously. Using a higher frame rate would result in more data points and further reduce the error.

The nurdled wink results are consistent with the findings in "Tiddlywinks Photography". In his Photo 1 and Slide 3/20, Tucker found that a nurdled wink performed a half rotation in 1/24s and 1/30s, respectively. This is equal to 75 rad/s and 94 rad/s, respectively, and our result of 77.21 ± 2.56 rad/s lies within this range. One limitation of Tucker's work is that it does not measure the angular velocity of winks that are not nurdled, which would provide useful data for comparison with our results and nurdled winks.

One possible explanation for why the angular velocity of a nurdled wink is less than that of a far wink is the amount of torque that must be applied to pot the wink. A farther wink would require more torque and thus result in a greater angular velocity. However, slight variations in the angle of the moment arm, which is determined by the squidger angle at the time of release, could also affect the results. It is difficult to determine if the small difference we found in angular velocities was due to torque differences or human inconsistency without a mechanical potting device. Such a machine could be constructed in the future to aid analysis.

5. Conclusions and Recommendations

The purpose of this experiment was to measure and compare the angular velocities of a nurdled wink and far wink. We found that the angular velocity of a far wink was greater than that of a nurdled wink when potting. The angular velocities were 83.69 ± 2.46 rad/s and 77.21 ± 2.56 rad/s, respectively. Although both cases demonstrated a negative mean value for acceleration, they were not statistically significant from zero, indicating only minimal deceleration.

Our findings are consistent with Tucker's measurements of the angular velocity of nurdled winks. Utilizing frame rates greater than 500fps and more lighting could also decrease the error in the angular acceleration measurements. An even simpler adjustment would be to use a white background and wear a white-sleeved shirt. Finally, the construction of a mechanical potting device would be extremely useful for the purpose of repeatability.

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References

[1] R. Tucker, "Tiddlywinks Photography", 6.163 Strobe Project Laboratory Project Report, MIT, Fall, 1979 (unpublished).

[2] "The Official Rules of Tiddlywinks", English Tiddlywinks Association, April, 2005 (www.etwa.org).